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The Stability Analysis of Manufacturer-Stackelberg Process in Two-Echelon Supply-Chain

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Abstract

In the two-echelon supply-chain literature, the manufacturer-Stackelberg process, i.e. the manufacturer is a Stackelberg leader and the retailer is a Stackelberg follower, is one of the most common gaming assumptions. In which, the optimal prices and optimal profits of the manufacturer and the retailer should be affected by the shape of the demand curve. In this paper, we discuss the influence to the optimal prices and profits by changing the parameters in demand curve, and analyze the stability of the optimal prices and profits of the manufacturer and the retailer. By using the Lipschitz properties of the optimal price-functions and profit-functions on the parameters of the demand curve, we find that there are sizable distinction in the stability of the optimal prices and profits of the manufacturer and the retailer with different demand curves.

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1. Introduction

The relationship between the manufacturer and the retailer has been investigated by many researchers. The increasing attention to supply chains leads to more studies on mulitiple-echelon systems [1-5].

In the two-echelon supply-chain literature, the manufacturer-Stackelberg process (abbreviated as m-St), i.e. the manufacturer is a Stackelberg leader and the retailer is a Stackelberg follower, is one of the most common gaming assumptions.

In this paper, we will discuss how the changing of parameters in a demand curve affects the optimal prices and profits of the manufacturer and the retailer. And farther, we analyze the stability of the optimal solutions in the two-echelon supply-chain process.

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2. Preparation knowledge

In this section, some symbols and background materials on the stability in mathematical programming and Lipschitz property of functions are given.

2.1. Basic definitions

In this section, some symbols are given for convenience, and the form of these symbols according to Lau-Lau [3].

Nomenclature

Π Profit. Π may have two subscripted letters and two superscripted letters, e.g., $\Pi_{lR}^{[mS]}$. Π 's two-lettered superscript "$[mS]$" designates for m-St process. Π 's first subscripted letter denotes the demand-curve form; it will be either l for "linear", c for "iso-elastic" or e for "exponen-tial". Π 's second subscripted letter will be either M for manufacture, or R for retailer.

c unit manufacturing cost incurred by the manufacturer.

w unit wholesale price charged to the retailer by the manufacturer.

p unit retail price the retailer charges the consumers.

D_p demand function of p. And D_p 's second subscript denotes different type of the demand function, e.g., D_{pl}.

There are three kinds of demand functions considered in this paper:

- First kind is the linear demand curve $D_{pl} = a - bp$, where a and b are positive parameters, $a/b \geq p$.
- Second kind is the iso-elastic curve $D_{pl} = Kp^{-\alpha}$, where K and α are positive parameters, $\alpha > 1$.
- Third kind is the exponential curve $D_{pl} = \gamma e^{-\beta p}$, where γ and β are positive parameters.

In this paper, we consider the case of one manufacturer selling to one retailer, so their profit functions without logistic cost components are, respectively:

$$\Pi_M = (w-c)D_p,\ \Pi_R = (p-w)D_p \tag{1}$$

So the mathematical model of the m-St process can be described as a bilevel programming problem as follows:

$$\max_{w\geq 0}\left\{(w-c)D_p : p \in \arg\max_{p\geq 0}(p-w)D_p\right\} \tag{2}$$

2.2. Preparation knowledge

In this section, some background material on the stability in mathematical programming and Lipschitz property of functions which will be used later. We give only concise definitions and facts that will be needed in the paper, and there are more detailed information in [6-8].

For the parameter programming problem

$$\max\left\{f(x,t) : x \in S(t) \subseteq \Re^n, t \in T \subseteq \Re^m\right\} \tag{3}$$

We denote R_t^*, f_t^* as the optimal region and the optimal value of problem (3), where t is a parameter. Then we have some definitions on the stability of problem (3).

Definition 1 Problem (3) is called weak stable on parameter $t_0 \in T$, if

1) $R_{t_0}^* \neq \Phi$,

2) $\forall \varepsilon > 0$, $\exists \alpha > 0$, when $\|t - t_0\| \le \alpha$ for $t \in T$, there is $|f(t) - f(t_0)| < \varepsilon$.

Definition 2 Problem (3) is called strong stable on parameter $t_0 \in T$, if

1) problem (3) is weak stable on parameter $t_0 \in T$,

2) for all $t \in T$ which satisfy $\|t - t_0\| \le \alpha$, there are $R_t^* \subseteq [R_{t_0}^*]_{+\varepsilon}$ and $R_{t_0}^* \subseteq [R_t^*]_{+\varepsilon}$, where $[S]_{+\varepsilon} = \bigcup_{x \in S} \{y : \|x - y\| < \varepsilon\}$.

Proposition 1 If there is only one element in R_t^* for all parameter $t \in T$, the weak stability is equivalent to the strong stability for problem (3).

Definition 3 Function $f(x): \Re^n \to \Re^1$ is called Lipschitz in $I \subseteq \Re^n$, if there exist a constant $L \in \Re_+^1$, for any $x_1, x_2 \in I$, there is $|f(x_1) - f(x_2)| \le L|x_1 - x_2|$.

Proposition 2 If a smooth function $f(x)$ has limitary differential coefficient in I, then it is Lipschitz in I.

3. Stability analysis in the M-St process

For the retailer is a Stackelberg follower in problem (2), so we can solve the retailer's optimal retail price $p^{[mS]*}$ firstly with parameter w, and then introduce $p^{[mS]*}$ into the manufacturer's decision, we will get the optimal wholesale price $w^{[mS]*}$, and the optimal profits $\Pi_M^{[mS]*}$ and $\Pi_R^{[mS]*}$.

For the special structure in problem (2), we get that the optimal solutions and values in which will be primary functions of parameters in the demand curve. Thereby, the stability of problem (2) can translate into the Lipschitz property of the objective function.

In this section, the main works are to analyze the stabilities of the optimal prices and profits in the m-St press, with the three kinds of demand curves mentioned in Section 2.

3.1. Linear demand curve

Lau-Lau [2] give the optimal prices and profits in the m-St press with linear demand curve as follows:

$$w^{[mS]*} = \frac{a + bc}{2b},\ p^{[mS]*} = \frac{3a + bc}{4b},\ \Pi_M^{[mS]*} = \frac{a + bc^2}{8b},\ \Pi_R^{[mS]*} = \frac{a + bc^2}{16b},\ \Pi_M^{[mS]*} \Big/ \Pi_R^{[mS]*} = 2. \tag{4}$$

First, we consider the influence of parameter a in the optimal prices and profits.

Theorem 1 The optimal prices of the manufacturer and the retailer in m-St process are stable to demand curve parameter a in a linear demand curve.

Proof: For any given parameter $b > 0$, the optimal prices $w^{[mS]*}$ and $p^{[mS]*}$ are both linear functions of parameter a with constant slopes:

$$\frac{\partial w^{[mS]*}}{\partial a} = \frac{1}{2b},\ \frac{\partial p^{[mS]*}}{\partial a} = \frac{3}{4b}. \tag{5}$$

So according to Definition 2, we know that $w^{[mS]*}$ and $p^{[mS]*}$ are Lipschitz smooth functions on $a \in (0, +\infty)$.

Then it illuminates that the optimal prices of the manufacturer and the retailer are stable to demand curve parameter a. □

And Theorem 1 means that, if the linear demand curve just budges without changing the shape, the optimal prices of the manufacturer and the retailer will only change tiny corresponding.

Theorem 2 The stability of the optimal profits of the manufacturer and the retailer in m-St process to parameter a weaken in a linear demand curve while a augments to $+\infty$.

Proof: Since $c < a / b$ and

$$\frac{\partial \Pi_M^{[mS]*}}{\partial a} = \frac{a - bc}{4b} > 0,\ \frac{\partial \Pi_R^{[mS]*}}{\partial a} = \frac{a - bc}{8b} > 0. \tag{6}$$

Then $\Pi_M^{[mS]*}$ and $\Pi_R^{[mS]*}$ are strictly monotone increasable smooth functions of parameter a. Moreover,

$$\frac{\partial \Pi_M^{[mS]*}}{\partial a} \to +\infty, \frac{\partial \Pi_R^{[mS]*}}{\partial a} \to +\infty, \text{ when } a \to +\infty. \tag{7}$$

Hereby, $\Pi_M^{[mS]*}$ and $\Pi_R^{[mS]*}$ are not Lipschitz functions on $a \in (0, +\infty)$, but are Lipschitz smooth functions on $a \in (0, A)$ for arbitrary positive number A.

So the optimal profits in m-St process are stable to demand curve parameter a while a is far from $+\infty$. □

From the above two theorems, we can find that, to the same change in parameter a, there are quite different appearances in the corresponding change in the optimal prices and profits. The reason is that the same change of parameter a at different values will lead to different changes in the optimal output, and the optimal profits will be influenced.

And then we consider the influence of parameter b in the optimal prices and profits similarly.

Theorem 3 The optimal prices of the manufacturer and the retailer in m-St process are stable on parameter b in a linear demand curve, as long as b is not close to 0.

Proof: For any given parameter $a > 0$, there are

$$\frac{\partial w^{[mS]*}}{\partial b} = \frac{-a}{2b^2} < 0, \frac{\partial p^{[mS]*}}{\partial b} = \frac{-3a}{4b^2} < 0. \tag{8}$$

So $w^{[mS]*}$ and $p^{[mS]*}$ are strictly monotone decreasing smooth functions of parameter b. And

$$\frac{\partial w^{[mS]*}}{\partial b} \to 0, \frac{\partial p^{[mS]*}}{\partial b} \to 0, \text{ when } b \to +\infty; \frac{\partial w^{[mS]*}}{\partial b} \to -\infty, \frac{\partial p^{[mS]*}}{\partial b} \to -\infty, \text{ when } b \to 0^+. \tag{9}$$

So $w^{[mS]*}$ and $p^{[mS]*}$ are non-differential at $b = 0$ and Lipschitz functions on $b \in [B_1, +\infty)$ for arbitrary positive number B_1.

Therefore the optimal prices of the manufacturer and the retailer are stable on parameter b, as long as b is not close to 0. □

And the above fact in Theorem 3 can be illuminated by the price elasticity theory. From the definition of price elasticity coefficient E_D of the demand curve, we know that

$$E_{D_{pl}} = \frac{-bp}{a - bp}. \tag{10}$$

And when b tend to 0, $E_{D_{pl}}$ will become smaller and the shape of demand curve will become more cliffier, then the sensitivity of the optimal price to parameter b is higher, shown as Fig 1.

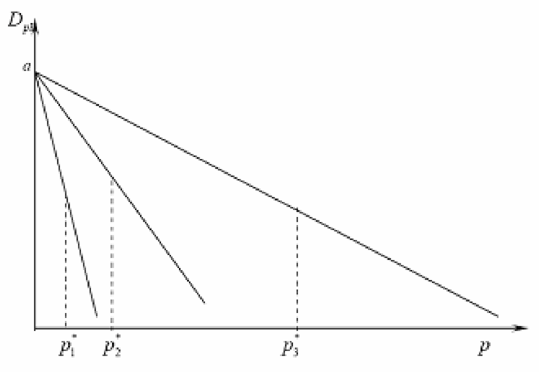

Fig. 1. the linear demand curve

Similarly to Theorem 3, we have

Theorem 4 The optimal profits of the manufacturer and the retailer in m-St process are stable on parameter b in a linear demand curve, as long as b is not close to 0.

Proof: Since

$$\frac{\partial \Pi_M^{[mS]*}}{\partial b} = \frac{-(a-bc)(a+bc)}{8b^2} < 0 \,, \frac{\partial \Pi_R^{[mS]*}}{\partial a} = \frac{-(a-bc)(a+bc)}{16b^2} < 0 \,. \tag{11}$$

Then $\Pi_M^{[mS]*}$ and $\Pi_R^{[mS]*}$ are strictly monotone descending smooth functions of parameter b.

However there are

$$\frac{\partial \Pi_M^{[mS]*}}{\partial b} \to -\infty \,, \frac{\partial \Pi_R^{[mS]*}}{\partial b} \to -\infty \,, \; when \; b \to 0^+ \,; \; \frac{\partial \Pi_M^{[mS]*}}{\partial b} \to \frac{c^2}{8} \,, \frac{\partial \Pi_R^{[mS]*}}{\partial b} \to \frac{c^2}{16} \,, \; when \; b \to +\infty \,. \tag{12}$$

So the optimal profits of the manufacturer and the retailer are stable on parameter b, as long as b is not close to 0.

3.2. Iso-elastic demand curve

The optimal prices and profits in the m-St press with iso-elastic demand curve are

$$w^{[mS]*} = \frac{\alpha c}{\alpha - 1} \,, \; p^{[mS]*} = \frac{\alpha^2 c}{(\alpha-1)^2} \,, \; \Pi_M^{[mS]*} = \frac{K(\alpha-1)^{2\alpha-1}}{\alpha^{2\alpha} c(\alpha-1)} \,, \; \Pi_R^{[mS]*} = \frac{K(\alpha-1)^{2(\alpha-1)}}{\alpha^{2\alpha-1} c(\alpha-1)} \,. \tag{13}$$

And the analyses of the curve parameters in an iso-elastic demand curve are studied as follows.

Theorem 5 The optimal prices and profits of the manufacturer and the retailer are stable to demand curve parameter K in an iso-elastic demand curve.

Proof: For any given parameter $\alpha > 1$, there are

$$\frac{\partial w^{[mS]*}}{\partial K} = \frac{\partial p^{[mS]*}}{\partial K} = 0 \,. \tag{14}$$

So there are no change in $w^{[mS]*}$ and $p^{[mS]*}$ when parameter K has some alteration. And since

$$\frac{\partial \Pi_M^{[mS]*}}{\partial K} = \frac{(\alpha-1)^{2\alpha-1}}{\alpha^{2\alpha} c(\alpha-1)} \,, \frac{\partial \Pi_R^{[mS]*}}{\partial K} = \frac{(\alpha-1)^{2(\alpha-1)}}{\alpha^{2\alpha-1} c(\alpha-1)} \,. \tag{15}$$

So the optimal profits of the manufacturer and the retailer are stable to demand curve parameter K.

And now we discuss the influence of parameter α.

Theorem 6 The optimal prices of the manufacturer and the retailer in m-St process are stable on parameter α in an iso-elastic demand curve, as long as α is not close to 1.

Proof: First, for

$$\frac{\partial w^{[mS]*}}{\partial \alpha} = \frac{-c}{(\alpha-1)^2} < 0 \,, \frac{\partial p^{[mS]*}}{\partial \alpha} = \frac{-2c\alpha}{(\alpha-1)^3} < 0 \,. \tag{16}$$

So $w^{[mS]*}$ and $p^{[mS]*}$ are strictly monotone decreasing smooth functions of parameter α. And

$$\frac{\partial w^{[mS]*}}{\partial \alpha} \to 0 \,, \frac{\partial p^{[mS]*}}{\partial \alpha} \to 0 \,, \; when \; \alpha \to +\infty \,; \; \frac{\partial w^{[mS]*}}{\partial \alpha} \to -\infty \,, \frac{\partial p^{[mS]*}}{\partial \alpha} \to -\infty \,, \; when \; \alpha \to 1^+ \,. \tag{17}$$

So the optimal prices of the manufacturer and the retailer are unstable to parameter α, when α is close to 1, and $w^{[mS]*}$ and $p^{[mS]*}$ are non-differential at $\alpha = 1$.

Therefore $\forall \varepsilon > 0$, $w^{[mS]*}$ and $p^{[mS]*}$ are Lipschitz smooth functions for $\alpha \in [1+\varepsilon, +\infty)$, and they are stable to α in this region. □

Theorem 7 The optimal profit of the manufacturer is stable to parameter α in an iso-elastic demand curve, however the optimal profit of the retailer is unstable to parameter α when α approaches to 1.

Proof: For

$$\frac{\partial \Pi_M^{[mS]*}}{\partial \alpha} = K \frac{(\alpha-1)^{2\alpha-1}[2\ln(\alpha-1) + \frac{1}{\alpha-1} - \ln c\alpha]}{\alpha^{2\alpha} c^{\alpha-1}} > 0. \tag{18}$$

Then $\Pi_M^{[mS]*}$ is strictly monotone increasing smooth functions of parameter $\alpha > 1$. And

$$\frac{\partial \Pi_M^{[mS]*}}{\partial \alpha} \to 1, \textit{ when } \alpha \to 1^+; \ \frac{\partial \Pi_M^{[mS]*}}{\partial \alpha} \to 0, \textit{ when } \alpha \to +\infty. \tag{19}$$

Thereby, the optimal profit of the manufacturer is stable to parameter α. Whereas, since

$$\frac{\partial \Pi_P^{[mS]*}}{\partial \alpha} = K \frac{(\alpha-1)^{2\alpha-2}[2\ln(\alpha-1) + \frac{1}{\alpha-1} - \ln c\alpha^2]}{\alpha^{2\alpha-1} c^{\alpha-1}} < 0. \tag{20}$$

Then $\Pi_P^{[mS]*}$ is strictly monotone decreasing smooth functions of parameter $\alpha > 1$. And

$$\frac{\partial \Pi_P^{[mS]*}}{\partial \alpha} \to -\infty, \textit{ when } \alpha \to 1^+; \ \frac{\partial \Pi_P^{[mS]*}}{\partial \alpha} \to 0, \textit{ when } \alpha \to +\infty. \tag{21}$$

Thereby, the optimal profit of the retailer is unstable to parameter α when α approaches to 1.

3.3. Exponential demand curve

The optimal prices and profits in the m-St press with exponential demand curve are

$$w^{[mS]*} = \frac{\beta c + 1}{\beta}, \ p^{[mS]*} = \frac{\beta c + 2}{\beta}, \ \Pi_M^{[mS]*} = \Pi_R^{[mS]*} = \frac{\gamma e^{-\beta p}}{\beta}. \tag{22}$$

First for the optimal prices of the manufacturer and the retailer are independent of the value of parameter γ, and

$$\frac{\partial \Pi_M^{[mS]*}}{\partial \gamma} = \frac{e - \beta p}{\beta} \tag{23}$$

is a constant to γ, so the influence of parameter γ to the optimal profits and prices is very small.

Theorem 8 The optimal profits and prices of the manufacturer and the retailer are unstable to parameter β in an exponential demand curve when β approaches to 0.

Proof: There are

$$\frac{\partial w^{[mS]*}}{\partial \beta} = \frac{-1}{\beta^2}, \ \frac{\partial p^{[mS]*}}{\partial \beta} = \frac{-2}{\beta^2}. \tag{24}$$

And

$$\frac{\partial w^{[mS]*}}{\partial \beta} \to -\infty, \ \frac{\partial p^{[mS]*}}{\partial \beta} \to -\infty, \textit{ when } \beta \to 0^+; \ \frac{\partial w^{[mS]*}}{\partial \beta} \to 0, \ \frac{\partial p^{[mS]*}}{\partial \beta} \to 0, \textit{ when } \beta \to +\infty. \tag{25}$$

Then the optimal prices of the manufacturer and the retailer are unstable to parameter β, when β is close to 0.

Further, for

$$\frac{\partial \Pi_M^{[mS]*}}{\partial \beta} = \frac{\partial \Pi_R^{[mS]*}}{\partial \beta} = \gamma e^{-\beta p} \frac{-\beta p - 1}{\beta^2} < 0 \qquad (26)$$

And

$$\frac{\partial \Pi_M^{[mS]*}}{\partial \beta} \to -\infty, \textit{ when } \beta \to 0^+; \ \frac{\partial \Pi_M^{[mS]*}}{\partial \beta} \to 0, \textit{ when } \beta \to +\infty. \qquad (27)$$

Then the optimal profits of the manufacturer and the retailer are unstable to parameter β when β approaches to 0. □

4. Conclusions

In this paper, we discuss the influence to the optimal prices and profits by changing the parameters in three different demand curves in the manufacturer-Stackelberg process of two-echelon supply-chain. And we find that there are sizable distinction in the stability of the optimal prices and profits of the manufacturer and the retailer with different demand curves.

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